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Question Paper Code: E3122

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2010

Fourth Semester

Electronics and Communication Engineering

MA2261 — PROBABILITY AND RANDOM PROCESS

(Common to Bio-Medical Engineering)

(Regulation 2008)

Time: Three hours — Maximum: 100 Marks

Use of Statistical Tables is permitted

Answer ALL Questions

PART A — (10 × 2 = 20 Marks)

1. If the p.d.f. of a random variable X is $f(x)=\frac{x}{2}$ in $0\le x\le 2$, find $P(X>1.5/X>1)$.

2. If the MGF of a uniform distribution for a random variable X is $\frac{1}{t}\left(e^{5t}-e^{4t}\right)$, find $E(X)$.

3. Find the value of k, if $f(x,y)=k(1-x)(1-y)$ in $0<x,y<1$ and $f(x,y)=0$, otherwise, is to be the joint density function.

4. A random variable X has mean 10 and variance 16. Find the lower bound for $P(5<X<15)$.

5. Define a wide sense stationary process.

6. Define a Markov chain and give an example.

7. Find the mean of the stationary process $\{x(t)\}$, whose autocorrelation function is given by $R(\tau)=16+\frac{9}{1+16\tau^2}$.

8. Find the power spectral density function of the stationary process whose autocorrelation function is given by $e^{-|\tau|}$.

9. Define time-invariant system.

10. State autocorrelation function of the white noise.

PART B — (5 × 16 = 80 Marks)

11. (a) (i) The probability mass function of random variable X is defined as $P(X=0)=3C^2$, $P(X=1)=4C-10C^2$, $P(X=2)=5C-1$, where $C>0$, and $P(X=r)=0$ if $r \neq 0,1,2$. Find

(1) The value of C

(2) $P(0<X<2/x>0)$

(3) The distribution function of X

(4) The largest value of X for which $F(x)<\frac{1}{2}$. (8)

(ii) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8. What is the probability that he will finally pass the test

(1) On the fourth trial and

(2) In less than 4 trials? (8)

Or

(b) (i) Find the MGF of the two parameter exponential distribution whose density function is given by $f(x)=\lambda e^{-\lambda(x-a)}$, $x \geq a$ and hence find the mean and variance. (8)

(ii) The marks obtained by a number of students in a certain subject are assumed to be normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that two of them will have marks over 70?(8)

12. (a) (i) For the bivariate probability distribution of (X,Y) given below :

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Find the marginal distributions, conditional distribution of X given $Y=1$ and conditional distribution of Y given $X=0$. (8)

(ii) Find the covariance of X and Y, if the random variable (X,Y) has the joint p.d.f. $f(x,y)=x+y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $f(x,y)=0$, otherwise. (8)

Or

(b) (i) The joint p.d.f. of two dimensional random variable (X,Y) is given by $f(x,y)=\frac{8}{9}xy$, $0\le x\le y\le 2$ and $f(x,y)=0$, otherwise. Find the densities of X and Y, and the conditional densities $f(x/y)$ and $f(y/x)$. (8)

(ii) A sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using Central Limit Theorem, find the probability with which the mean of the sample will not differ from 60 by more than 4. (8)

13. (a) (i) Examine whether the random process $\{X(t)\}=A\cos(\omega t+\theta)$ is a wide sense stationary if A and ω are constants and θ is uniformly distributed random variable in $(0,2\pi)$. (8)

(ii) Assume that the number of messages input to a communication channel in an interval of duration t seconds, is a Poisson process with mean $\lambda=0.3$. Compute

(1) The probability that exactly 3 messages will arrive during 10 second interval

(2) The probability that the number of message arrivals in an interval of duration 5 seconds is between 3 and 7. (8)

Or

(b) (i) The random binary transmission process $\{X(t)\}$ is a wide sense process with zero mean and autocorrelation function $R(\tau)=1-\frac{|\tau|}{T}$, where T is a constant. Find the mean and variance of the time average of $\{X(t)\}$ over $(0,T)$. Is $\{X(t)\}$ mean-ergodic? (8)

(ii) The transition probability matrix of a Markov chain $\{X_n\},n=1,2,3,...$ having three states 1, 2, 3 is $P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$, and the initial distribution is $P^{(0)}=[0.7\ \ 0.2\ \ 0.1]$, Find $P(X_2=3)$ and $P(X_3=2,\ X_2=3,\ X_1=3,\ X_0=2)$. (8)

14. (a) (i) Find the autocorrelation function of the periodic time function $\{X(t)\}=A\sin\omega t$. (8)

(ii) The autocorrelation function of the random binary transmission $\{X(t)\}$ is given by $R(\tau)=1-\frac{|\tau|}{T}$ for $|\tau|<T$ and $R(\tau)=0$ for $|\tau|<T$. Find the power spectrum of the process $\{X(t)\}$. (8)

Or

(b) (i) $\{X(t)\}$ and $\{Y(t)\}$ are zero mean and stochastically independent random processes having autocorrelation functions $R_{XX}(\tau) = e^{-|\tau|}$ and $R_{YY}(\tau) = \cos 2\pi\tau$ respectively. Find

(1) The autocorrelation function of $W(t) = X(t) + Y(t)$ and $Z(t) = X(t) - Y(t)$

(2) The cross correlation function of $W(t)$ and $Z(t)$. (8)

(ii) Find the autocorrelation function of the process $\{X(t)\}$ for which the power spectral density is given by $S_{XX}(\omega) = 1 + \omega^2$ for $|\omega| < 1$ and $S_{XX}(\omega) = 0$ for $|\omega| > 1$. (8)

15. (a) (i) A wide sense stationary random process $\{X(t)\}$ with autocorrelation $R_{XX}(t) = e^{-a|\tau|}$ where A and a are real positive constants, is applied to the input of an Linear transmission input system with impulse response $h(t) = e^{-bt}u(t)$ where b is a real positive constant. Find the autocorrelation of the output $Y(t)$ of the system. (8)

(ii) If $X(t)$ is a band limited process such that $S_{XX}(\omega) = 0$ when $|\omega| > \sigma$, prove that $[2R_{XX}(0) - R_{XX}(\tau)] \leq \sigma^2 \tau^2 R_{XX}(0)$. (8)

Or

(b) (i) Assume a random process $X(t)$ is given as input to a system with transfer function $H(\omega) = 1$ for $-\omega_0 < \omega < \omega_0$. If the autocorrelation function of the input process is $\frac{N_0}{2}\delta(t)$, find the autocorrelation function of the output process. (8)

(ii) If $Y(t) = A\cos(\omega t + \theta) + N(t)$, where A is a constant, θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and $\{N(t)\}$ is a band limited Gaussian white noise with a power spectral density $S_{NN}(\omega) = \frac{N_0}{2}$ for $|\omega - \omega_0| < \omega_B$ and $S_{NN}(\omega) = 0$, elsewhere. Find the power spectral density of $Y(t)$, assuming that $N(t)$ and θ are independent. (8)
